1 Find the derivative of the following functions:

a

$$y = 10 - e^{-0.5x}$$
$$y' = 0.5e^{-0.5x}$$

Exponential and chain rule

b

$$y = \ln(1 + x)$$
$$y' = \frac{1}{1 + x}$$

Logarithm

c

$$y = x^{0.7}$$
$$y' = 0.7x^{-0.3}$$
$$y' = \frac{0.7}{x^{0.3}}$$

Power rule

d

$$y = x^{1.7}$$
$$y' = 1.7x^{0.7}$$

Power rule

e

$$y = 3 + 5x - 6x^2$$
$$y' = 5 - 12x$$

Sum and power

f

$$y = (3 + 5x - 6x^2)^2$$
$$y' = 2(5 - 12x)(3 + 5x - 6x^2)$$

Sum, power, and chain rule

g

$$y = 5e^{3+5x-6x^2}$$
$$y' = 5e^{3+5x-6x^2}(5 - 12x)$$

Exponential, sum, power, and chain rule

h

$$
\begin{aligned}
y &= 5\ln(3+5x-6x^2) \\
y' &= \frac{5}{3+5x-6x^2}(5-12x) \\
y' &= \frac{25-60x}{3+5x-6x^2}
\end{aligned}
$$

Logarithm, sum, power, and chain rule

i

$$y = x^2 e^{0.5x}$$

$$f(x) = x^2 \qquad g(x) = e^{0.5x}$$

$$
\begin{aligned}
\frac{df(x)g(x)}{dx} &= f(x)g(x)' + f(x)'g(x) \\
&= x^2(0.5e^{0.5}) + 2xe^{0.5x} \\
&= e^{-.5x}(0.5x^2+2x) \\
&= e^{0.5x}(0.5x+2)x
\end{aligned}
$$

Product rule

2 Check for the monotonicity of functions (a-e) in question 1, assuming $x \geq 0$.

$$
\begin{aligned}
y &= 10 - e^{-0.5x} \\
y' &= 0.5e^{-0.5x}
\end{aligned}
$$

Since $y' = 0.5e^{-0.5x} > 0$ $(x \geq 0)$, this equation $10 - e^{-0.5x}$ is monotonically increasing.

b

$$
\begin{aligned}
y &= \ln(1+x) \\
y' &= \frac{1}{1+x}
\end{aligned}
$$

Since $y' = \frac{1}{1+x} > 0$ $(x \geq 0)$, this equation $\ln(1+x)$ is monotonically increasing.

c

$$
\begin{aligned}
y &= x^{0.7} \\
y' &= 0.7x^{-0.3} \\
y' &= \frac{0.7}{x^{0.3}}
\end{aligned}
$$

Since $y' = \frac{0.7}{x^{0.3}} > 0$ $(x \geq 0)$, this equation $x^{0.7}$ is monotonically increasing.

d

$$y = x^{1.7}$$
$$y' = 1.7x^{0.7}$$

Since $y' = 1.7x^{0.7} > 0$ $(x \geq 0)$, this equation $x^{1.7}$ is monotonically increasing.

e

$$y = 3 + 5x - 6x^2$$
$$y' = 5 - 12x$$

Since $5 - 12x$ is positive from $x = [0, \frac{5}{12})$ but negative from $x = (\frac{5}{12}, \infty)$, this equation $3 + 5x - 6x^2$ is non-monotonic overall. Or we can also say the function is monotonically increasing from the domain of $x = [0, \frac{5}{12})$ and monotonically decreasing from the domain of $x = (\frac{5}{12}, \infty)$.

3 Check for the convexity of functions (a-e) in question 1, assuming $x \geq 0$.

a

$$y = 10 - e^{-0.5x}$$
$$y' = 0.5e^{-0.5x}$$
$$y'' = -0.25e^{-0.5x}$$
$$y'' = \frac{-0.25}{e^{0.5x}}$$

Since $y'' = \frac{-0.25}{e^{0.5x}} < 0$ $(x \geq 0)$, this equation $10 - e^{-0.5x}$ is strictly concave.

b

$$y = \ln(1 + x)$$
$$y' = \frac{1}{1 + x}$$
$$y'' = \frac{1}{(1 + x)^2}$$

Since $y'' = \frac{1}{(1 + x)^2} > 0$ $(x \geq 0)$, this equation $y = \ln(1 + x)$ is strictly convex.

c

$$y = x^{0.7}$$
$$y' = 0.7x^{-0.3}$$
$$y' = \frac{0.7}{x^{0.3}}$$
$$y'' = -0.21x^{-1.3}$$
$$y'' = \frac{-0.21}{x^{1.3}}$$

Since $y'' = \frac{-0.21}{x^{1.3}} < 0$ $(x \geq 0)$, this equation $y = x^{0.7}$ is strictly concave.

d

$$y = x^{1.7}$$
$$y' = 1.7x^{0.7}$$
$$y'' = 1.19x^{-0.3}$$
$$y'' = \frac{1.19}{x^{0.3}}$$

Since $y'' = \frac{1.19}{x^{0.3}} > 0$ $(x \geq 0)$, this equation $y = x^{1.7}$ is strictly convex.

e

$$y = 3 + 5x - 6x^2$$
$$y' = 5 - 12x$$
$$y'' = -12$$

Since $y'' = -12 < 0$ $(x \geq 0)$, this equation $y = 3 + 5x - 6x^2$ is strictly concave.

4

a

In order to maximize yield in terms of x the amount of water in acre-inches, we must find when the slope of the production function is equal to zero.

$$y = 468.28 + 70.97x - 1.55x^2 \quad (1)$$
$$y' = 70.97 - 3.10x = 0 \quad (2)$$

Solving for x (the amount of water in acre-inches), we get:

$$3.10x = 70.97 \quad (1)$$
$$x = 22.89 \quad (2)$$

Therefore, in order to maximize yield, we would need to apply 22.98 acre-inches of water to the land.

b

Since we know $\pi = Py - wx$, we can set $\pi' = 0$ in order to maximize profit doing the following

$$\pi = Py - wx \quad (1)$$
$$= (1)(468.28 + 70.97x - 1.55x^2) - 10x \quad (2)$$
$$= 468.28 + 60.97x - 1.55x^2 \quad (3)$$

Now we can take the derivative of π and set that equal to zero in order to maximize profit

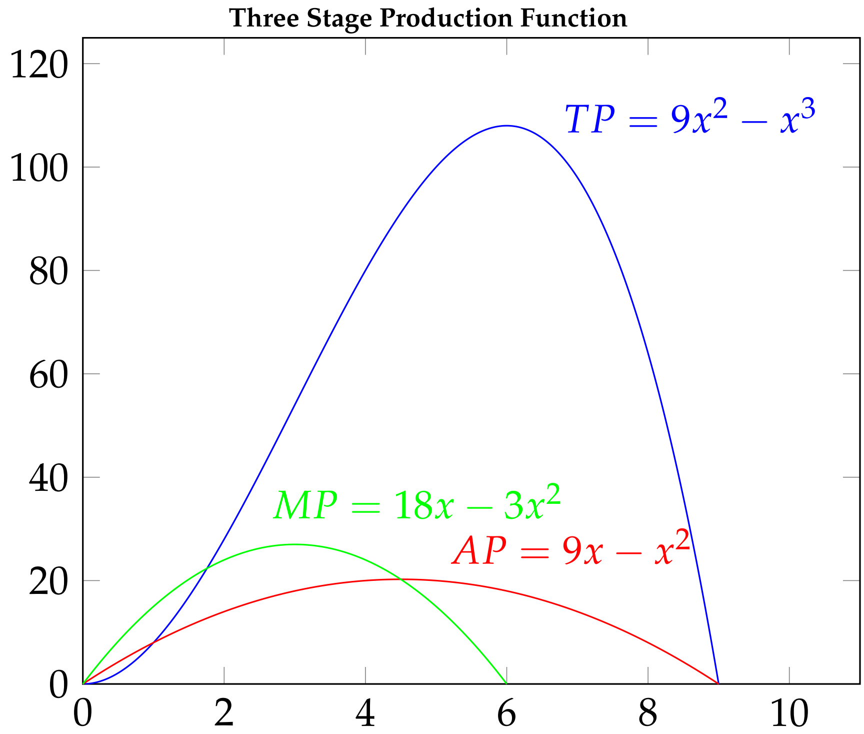
$$\pi = 468.28 + 60.97x - 1.55x^2 \quad (1)$$
$$\pi' = 60.97 - 3.10x = 0 \quad (2)$$
$$(3)$$

Solving for x in this instance gives us

$$60.97 - 3.10x = 0 \quad (1)$$
$$3.10x = 60.97 \quad (2)$$
$$x = 19.66 \quad (3)$$

Therefore, in order to maximize profit, we would need to apply 16.44 acre-inches of water to the land.

5



$$\pi = Py - wx \quad (1)$$
$$= (1)(9x^2 - x^3) - 24x \quad (2)$$
$$= 9x^2 - x^3 - 24x \quad (3)$$

To find maximized profit, we would again have to set the derivative of equation (3) to zero:

$$\pi = 9x^2 - x^3 - 24x \quad (1)$$
$$\pi' = 18x - 3x^2 - 24 = 0 \quad (2)$$
$$= -(3x^2 - 18x + 24) = 0 \quad (3)$$
$$= -(3x - 12)(x - 2) = 0 \quad (4)$$

Therefore, we get $x = 2$ and $x = 4$. We can plug these numbers back into our profit function (3) in order to compare which input quantity gives us higher profits.

$$\pi = 9(2^2) - 2^3 - 24(2) \qquad \pi = 9(4^2) - 4^3 - 24(4) \quad (1)$$
$$\pi = -20 \qquad \pi = -16 \quad (2)$$

We see that since $-16 > -20$, our profit is maximized when the input quantity is 4. However, since our profit is negative throughout the domain of $x = [0, \infty)$, our firm, in this problem, would likely not be producing since it would have no economic incentive to do so and the input quantity and output quantity would both be zero. It could also be noted that rather than maximizing profit in this problem, we are minimizing losses.